

Supersymmetric Dynamics of a Spin-1/2 Particle in an Extended External Field

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Abstract

We consider a electron in a external field in $D = 5$, through the Dirac equation in the Galilean symmetry approach, and in the Lorentz symmetry approach; from these we perform the nonrelativistic limit, then we proceed the supersymmetry of the same that is associated with the Galilean symmetry, we identify as a supersymmetry sector from the quantum-mechanical dynamics, and we got the algebra of fermionic charges. We naturally define as extra electrical vector E , and interpret the terms of energy coming from the fifth dimension. The energy from the fifth dimension, create this extra electrical vector E , associated with the fifth component of the external electrical field A , this makes the energy flow from the fifth dimension to the usual three-dimensional space, when some symmetries of the usual space are broken, giving a preferential direction in the space, even though the standard electrical and magnetic fields are null.

1 Introduction

Supersymmetry at low energies (non-relativistic limit) raises special interest for a great deal of reasons, [1], [2], [3], [4], [5]. If Supersymmetry is a symmetry of Nature, what we see today must be its low-energy remnant through the breaking realised by means of some mechanism [6], [7], [8]. In this limit, the underlying field theory should approach a Galilean invariant supersymmetric field theory and, by the Bargmann superselection rule [9], such a field theory should be equivalent to a supersymmetric Schrödinger equation in each particle number sector of the theory. One proposes a particle model based on the Dirac equation in $D=5$, but we show some detail about dimensions less than 5. This construction is embedded in a supersymmetric model consisting of an extended Wess-Zumino model with 2 matter chiral superfields. Analyzing this model in the low-energy limit, we proceed to the non-relativistic regime, dropping out

two degrees of freedom of the spinor that correspond to the weak component. The system is identified as a sector of Galilean invariant supersymmetric model. In some details this is equivalent work in the approach of Galilean symmetry or Lorentzean; We can move from one approach to another performing a linear transformation. In this way taking some conditions, the supersymmetry model can cover both approaches. Here after perform the supersymmetry, we show the interpretation of some parameters.

We can define an extra electrical field E , its origin is from the extra dimension, it is generated from the extra component of the electric vector A , in the direction of the extra dimension, even though A does not vary in the direction of this extra dimension. This extra electrical field E together with the presence of a charged particle with spin, for example an electron, will obviously break the symmetry of space even though the components of the electrical vector A , are zero in the usual space, ie not in the extra-dimension. This extra electrical field E in the presence of spin and charge like the electron, generates two mechanisms by which energy flows from the extra dimension to the usual space. The first mechanism is a correction of the Pauli-Dirac Hamiltonian of the order of A/m or E/m^2 , where m is mass, we call this term *Psch*; it will give non-zero contribution, if in the space there are a charged particle with spin-1/2 and a extra electrical vector E not constant at the usual space. The second mechanism that we call *Mtz* will give a correction of order A^2/m or E^2/m^3 since an electrical charge is in the presence of such extra electrical field E being constant or not in the usual space.

In this paper, we shall consider a particle of mass m and spin-1/2 in an external field, [10], [11], [12],[13]. We build up a superspace action for this model, read off the supersymmetry transformations of the component coordinates and then obtain the supersymmetric charge operators [14]. We analyse their algebra and pursue the investigation of the possible existence of a central charge in the system. Then we perform the interpretation of some results.

We set our discussion by first considering the case of (1+2) dimensions. The Dirac action in such a space-time may be written as

$$L = \bar{\Psi}(i\gamma^\mu D_\mu - m)\Psi,$$

where $\eta = (1, -1, -1)$ and $\gamma^0 = \sigma_3$, $\gamma^1 = i\sigma_2$, $\gamma^3 = \sigma_1$, $\gamma_3 = i\gamma_0\gamma_1$, $D_\mu \equiv \partial_\mu + ieA_\mu$. From the equation of motion,

$$(i\gamma^\mu D_\mu - m)\Psi = 0, \quad \text{where} \quad \Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix},$$

and with the approximation

$$E + m - e\phi \cong 2m, \quad \text{and} \quad \psi_2 \cong -\frac{(D_3 + iD_1)}{2m}\psi_1,$$

the Pauli Hamiltonian in 1+2 [15] takes over the form:

$$H = \frac{1}{2m} \vec{\nabla}^2 - \frac{ie}{2m} \vec{\nabla} \cdot \vec{A} - \frac{ie}{m} \vec{A} \cdot \vec{\nabla} - \frac{e^2}{2m} |\vec{A}|^2 - e\phi. \quad (1)$$

The minimal coupling does not generate here any coupling term of the type spin-magnetic field as it is the case for the Pauli Hamiltonian in D=1+3. We can also show that this coupling term cannot appear from a non-minimal coupling of the type:

$$D_\mu = \partial_\mu + ieA_\mu + ig\tilde{F}_\mu, \quad \text{where} \quad \tilde{F}_i = \frac{1}{2}\epsilon_{ijk}F^{jk}.$$

If we call 1+2, (t,x,y) the usual space and add a extra dimation z to 1+2, the dimension will be 1+3, (t,x,y,z), than we have in H at less two new terms related to corretions at Hamiltonian from the extra dimension, they are:

$$Psch = \frac{ie}{2m}\sigma^3\sigma^i\vec{\nabla}^i(\Omega), i = \{1, 2\} \quad \text{and} \quad Mtz = \frac{e^2}{2m}|\Omega|^2 \quad (1)$$

where Ω is A^3 , the extra component of the eletric vector A. In the same way if we look at the dimation 1+1, (t,x) as our usual space, and consider the direction z as an extra dimation,

$$Psch = \frac{ie}{2m}\sigma^3\sigma^1\vec{\nabla}^1(\Omega) \quad \text{and} \quad Mtz = \frac{e^2}{2m}|\Omega|^2 \quad (2)$$

In (1+3)D, we see that the bosonic degree of freedom, $x^i(t)$ (position), matches with the fermionic degree of freedom, $S^i(t)$ (spin). This makes possible the construction of superfields and then of a manifestally supersymmetric Lagrangian with linearly realised supersymmetry [14]. However, in (1+2)D, the same does not occur; angular momentum is not a vector any longer, the number of degreesof freedom of x^i and the spin components no longer match and therefore we cannot in this way build up superfields nor a supersymmetric Lagrangian in superspace. We can try to work in (1+3)D, starting from Dirac's equation, taking the non-relativistic limit and making a dimensional reduction to (1+2)D. But, we see that it is not possible to drag into (1+2)D the analogue of the coupling of spin-magnetic field that exists in (1+3)D [15]. We perform the supersymmetry in (1+4)D of this model starting from Dirac Equation in the approach of Lorentz and Galilean Symmetry where we may propose a match between space coordinates and the 5D spin, getting to an extended eletromagnetic-like field.

2 Pauli-Dirac in the Lorentz Approach, L-A

In the approach of Lorentz covariance we start from Dirac Lagrangean of a electron in a extended electromagnetic field, and we obtain the Pauli-Dirac Hamiltonian, the non-relativistic limit with minimal coupling.

We consider the following action in D=4+1,

$$L = \bar{\Psi}(i\Gamma^{\hat{\mu}}D_{\hat{\mu}} - m)\Psi; \quad (3)$$

where we define:

$$\mu \in \{0, 1, 2, 3, 4\}, \quad \eta = (+, -, -, -, -), \quad (4)$$

$$x^\mu = (t, x, y, z, w), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

$$D_\mu = \partial_\mu + ieA_\mu, \quad A_\mu = (A_0, A_i, \Lambda), \quad A^4 = \Omega = -\Lambda, \quad \{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu},$$

$$\partial_i \Longleftrightarrow \vec{\nabla}, \quad \partial_0 \Longleftrightarrow \frac{\partial}{\partial t}, \quad E^i \Longleftrightarrow \vec{E}, \quad B^i \Longleftrightarrow \vec{B},$$

$$F_{04} = \mathcal{E}, \quad F_{4i} = \mathcal{B}_i, \quad F_{ij} = -\epsilon_{ijk} B^k, \quad F_{0i} = E_i,$$

where $i, j, k \in \{1, 2, 3\}$. Notice that the scalar, \mathcal{E} , and the vector, $\vec{\mathcal{B}}$, accompany the vectors \vec{E} and \vec{B} , later on to be identified with the electrical and magnetic fields, respectively. Our explicit representation for the gamma-matrices is given in the Appendix 1. We set:

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix}, \quad \bar{\Psi} = \Psi^\dagger \Gamma^0$$

We now take the equation of motion for Ψ from the Euler-Lagrange equations (3). We suppose a stationary solution,

$$\Psi(x, t) = \exp(i\epsilon t)\chi(x);$$

here, we are considering that the external field does not depend on t , and $A_0 = 0$ and that $\partial_4(A_\mu) = 0$, then we obtain

$$\vec{E} \Longleftrightarrow F_{0i} \equiv \partial_0 A_i - \partial_i A_0 = 0; \quad (5)$$

$$\mathcal{E} \equiv F_{04} \equiv \partial_0 A_4 - \partial_4 A_0 = 0;$$

$$F_{ij} = \partial_i A_j - \partial_j A_i \equiv \epsilon_{ijk} B_k;$$

$$F_{i4} = \partial_i A_4 - \partial_4 A_i \equiv \partial_i \Lambda \equiv -\mathcal{B}_i.$$

Then, considering the non-relativistic limit of the reduced theory, the equations of motions read:

$$(\epsilon + m)I_2\chi_1 + (-i\sigma^i(\partial_i + ieA_i) + iI_2eA_4)\chi_2 = 0. \quad (6)$$

$$(i\sigma^i(\partial_i + ieA_i) + iI_2 eA_4)\chi_1 + (-\epsilon + m)I_2\chi_2 = 0.$$

therefore, we see that $\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$ loses two degrees of freedom, described by the "weak" spinor, χ_2 . So the Pauli-like Hamiltonian we get to reads as below:

$$H = \frac{1}{2m} \left[-\left(\vec{\nabla} - ie\vec{A}\right)^2 - e\vec{\sigma} \left(\vec{\nabla} \times \vec{A}\right) + e\vec{\sigma} \vec{\nabla}(A_4) + e^2(A_4)^2 \right], \quad (7)$$

A similar term that relates energy from the extra dimension, like at (2) and (1) is present here,

$$P_{sch} = -\frac{e}{2m} \sigma^i \vec{\nabla}^i(\Omega), \quad i = \{1, 2, 3\} \quad \text{and} \quad M_{tz} = \frac{e^2}{2m} |\Omega|^2 \quad (8)$$

where we used (4).

If we define, $p_i = -i\vec{\nabla}$ ($\hbar = 1$) and $\left(\vec{\nabla} \times \vec{A}\right)^k = \vec{B}^k$ then the Hamiltonian (7) will become

$$H = \frac{1}{2m} \left[(p_i - eA_i)^2 - eB^i \sigma^i - e\mathcal{B}_i \sigma^i + e^2 \Lambda^2 \right];$$

$$H = \frac{(p_i - eA_i)^2}{2m} + \frac{eB^i S_i}{m} - \frac{e\mathcal{B}_i S^i}{m} + \frac{e^2 \Lambda^2}{2m}; \quad \text{onde } S^i = \frac{\sigma^i}{2}; \quad (9)$$

the corresponding Lagrangian is given by:

$$L = \frac{m}{2} (\dot{x}_i)^2 + \frac{i}{2} \psi_i \dot{\psi}_i + eA_i \dot{x}_i + \frac{ie}{2m} B_i \epsilon_{ijk} \psi_j \dot{\psi}_k + \frac{ie}{2m} \mathcal{B}_i \epsilon_{ijk} \psi_j \dot{\psi}_k - \frac{e^2 \Lambda^2}{2m}, \quad (10)$$

where the dot is a derivative with respect to t. We can also write

$$L = \frac{m}{2} (\dot{x}_i)^2 + \frac{i}{2} \psi_i \dot{\psi}_i + eA_i \dot{x}_i + \frac{eB_i S_i}{m} - \frac{e\mathcal{B}_i S_i}{m} - \frac{e^2 \Lambda^2}{2m}, \quad (11)$$

where we identify the spin as the product below:

$$S_i = -\frac{i}{2} \epsilon_{ijk} \psi_j \dot{\psi}_k.$$

The spin (angular momentum) is built up from the bosonic, but it is of fermionic nature. In fact, we can check, with the help of the canonical anti-comutation relations for the fermions (these shall be written down later, in eq 20), that the algebra $[S_i, S_j] = i\epsilon_{ijk} S_k$ is satisfied.

3 Pauli-Dirac in the Galilean Approach, G-A

Here we show a brief review of the formalism of Galilean covariance and the Galilean Dirac Lagrangean of an electron in an extended electromagnetic field, also we will show the Galilean Pauli-Dirac Hamiltonian, the non-relativistic limit, with minimal and non-minimal coupling, when the spin is considered. More details can be found in [16],[17],[18]. We denote the coordinates in 4+1 dimensional flat space-time $x^\mu = (x^1, x^2, x^3, x^4, x^5)$ and choose the metric tensor by $\eta^{\mu\nu} = \text{diag}(-1, -1, -1, 1, -1)$, where $x^4 = t_r$ is the usual time and x^5 is the extra dimension. We now go to the light cone coordinates, where we denote the coordinates in 4+1 dimensional flat space-time $x^\mu = (x^1, x^2, x^3, t, s)$, by the linear transformation:

$$t = \frac{i}{\sqrt{2}}(x^4 - x^5) \text{ and } s = \frac{i}{\sqrt{2}}(x^4 + x^5) \quad (12)$$

and the inversion $x^4 = -\frac{i}{\sqrt{2}}(s + t)$ and $x^5 = -\frac{i}{\sqrt{2}}(s - t)$; In the light-cone coordinates, the metric tensor has the following form:

$$(g^{\mu\nu}) = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -1 & 0 \end{pmatrix}. \quad (13)$$

The set of all transformations that made $g^{\mu\nu}$ invariante is the Galilean Group, they are:

$$\begin{aligned} \mathbf{x}' &= R\mathbf{x} + \mathbf{v}t + \mathbf{a}, \\ t' &= t + b, \\ s' &= s + (R\mathbf{x}) \cdot \mathbf{v} + \frac{1}{2}\mathbf{v}^2t, \end{aligned} \quad (14)$$

where R is a 3×3 rotation matrix, \mathbf{v} is the relative velocity, \mathbf{a} and b are the space and time translations, respectively. We note that the transformation in Eq. (14) leaves invariant the scalar product $g_{\mu\nu}dx^\mu dx^\nu$. Now the relativistic energy-momentum $p_\mu = (\mathbf{p}, -\mathcal{E}, -m)$, the relationship for the massless particles $p^\mu p_\mu = -2p_s p_t + \vec{p}^2 = 0$ can be rewritten if we write $p_s = -m$ and $p_t = -\mathcal{E}$, as

$$\mathcal{E} = \frac{1}{2m} \vec{p}^2 \quad (4)$$

which is nothing but the expression of the kinetic energy for the non-relativistic particle with mass m in 4 dimensional space, 5 space-time dimensions. This is the natural interpretation of variables t and s , what is not more than the mix of the dimensions $x^4 = t_r$ usual time and x^5 ; that interpretation follow from the canonically conjugate variables: The five-momentum p_μ .

The Galilean Dirac equation

$$\gamma^\mu \partial_\mu \Psi = 0, \quad \bar{\Psi} \gamma^\mu \overleftarrow{\partial}_\mu = 0,$$

where $\gamma^\mu \partial_\mu = \gamma \cdot \nabla + \gamma^t \partial_t + \gamma^s \partial_s$ and the basic field is the spinor, $\Psi = \begin{pmatrix} \vartheta_+ \\ \vartheta_- \end{pmatrix}$, other notations about that section can be seen in appendix 1.

The Galilean-Dirac Lagrangian:

$$L = \frac{1}{2} \overline{\Psi} (\gamma^\mu \partial_\mu) \Psi - \frac{1}{2} \overline{\Psi} \left(\gamma^\mu \overleftarrow{\partial}_\mu \right) \Psi.$$

The Galilean-Dirac Lagrangian for a particle in a electromagnetic field can be obtained directly with the following definitions, the minimal coupling $p_\mu \rightarrow \pi_\mu = p_\mu - eA_\mu$, together another non-minimal coupling, $\pi_i \rightarrow \pi_i + C_i \eta$, where $\mathbf{C} = \frac{ie}{4m} \mathbf{E}$. This leads to $\mathbf{p} \rightarrow \mathbf{p} - e\mathbf{A} + ie \frac{\mathbf{E}}{4m} \eta$, $p_4 \rightarrow p_4 - eA_4$, $p_5 \rightarrow p_5 - eA_5$, where we use the 5-potential $A_\mu = (A_i, A_4, A_5) = (\mathbf{A}, -\phi_m, -\phi_e)$. The Galilean Pauli-Dirac Hamiltonian, the non-relativistic limit, with minimal and non-minimal coupling can be easily produced, this result is in [16].

Using here in G-A the same conditions used for the external field A in L-A, like $A^0 = 0$ in L-A will be $\phi_m = -\phi_e$ in G-A by transformations the (12). Also the electrical field we fix, $\mathbf{E} = \mathbf{0}$, giving us from [16] the following Hamiltonian to both, large and small components of Ψ ,

$$\mathcal{E} \vartheta_\pm = \left(\frac{(\mathbf{p} - e\mathbf{A})^2}{2m} + \frac{e}{m} \mathbf{S} \cdot \mathbf{B} \right) \vartheta_\pm. \quad (15)$$

That Hamiltonian is a sector that one in L-A, eq (9). We could imagine that in G-A it is not possible to represent the interactions from the fifth dimension in this way like we represent in G-A at eq (9), because the terms *Psch* and *Mtz* eq (8) that is responsible for this do not appear here, ie we have no interactions from the fifth dimension, but in the last section we will see that they can be generated with the inclusion of a dimensional factor in the non-minimal coupling even though the usual vector field vanishes, $\mathbf{E} = \mathbf{0}$, because we will be able to define any extra electrical field E generated by the energy from the extra-dimension.

If the usual electrical field E is not zero, we know from [16] that the Hamiltonian (15) in some conditions (like static field and others) has the following two terms of correction:

$$MR = \frac{e^2}{32m^3} \mathbf{E}^2 \quad \text{and} \quad OR = \frac{e}{8m^2} \nabla \cdot \mathbf{E} \quad (16)$$

4 N=1 Galilean Supersymmetry

To render our discussion more systematic, we think it is advisable to set up a superfield approach. We can define the N=1-supersymmetric model in analogy with the model presented above, eq.(11). We start defining the superfields by

$$\Phi_i(t, \theta) = x_i(t) + i\theta\psi_i(t) \quad \Sigma(t, \theta) = \xi(t) + \theta R(t), \quad (17)$$

$$\Lambda(x) = A_4(x) \quad \Lambda(\Phi_s) = \Lambda(x) + i(\partial_j \Lambda(x))\theta\psi_j.$$

The supercharge operators and the covariant derivatives are given by:

$$Q = \partial_\theta + i\theta\partial_t \quad D = \partial_\theta - i\theta\partial_t \quad H = i\partial_t. \quad (18)$$

Then, the N=1-supersymmetric Lagrangian that generates the Galilean Hamiltonian (15) can be written in superfields as:

$$\begin{aligned} \mathcal{L} = & \frac{i}{2}\dot{\Phi}_i D\Phi_i + ie(D\Phi_i)A_i(\Phi) + \frac{1}{2}\Sigma D\Sigma + \\ & -e\Sigma\Lambda(\Phi) + \frac{ie}{2}\epsilon_{ijk}\partial_i\Lambda(\Phi)\Phi_j D\Phi_k, \end{aligned} \quad (19)$$

where we have set $m = 1$.

The commutators and anticommutators for the superfield components are

$$[\psi_i, \psi_j]_+ = \delta_{ij}, \quad [\xi, \xi]_+ = -2i, \quad [x_i, p_j] = i\delta_{ij}, \quad (20)$$

$$[\psi_i, \xi]_+ = 0;$$

the other commutators vanish.

The component field R does not have dynamics; then, we use the equation of motion to remove it from the Lagrangian:

$$R = e\Lambda. \quad (21)$$

The supersymmetric action is then

$$\mathcal{S} = \int dt d\theta \mathcal{L}$$

We have written the supersymmetric Lagrangian in components, [14], under the form,

$$\mathcal{L} = K + \theta L$$

where we define K and L as follows:

$$\begin{aligned} K = & -\frac{1}{2}\dot{x}_i\psi_i - e\psi_i A^i + \frac{\xi R}{2} - e\xi\Lambda - \frac{e}{2}\epsilon_{ijk}\mathcal{B}_i x_j \psi_k \\ L = & \frac{1}{2}\dot{x}_i\dot{x}_i - \frac{1}{2}i\dot{\psi}_i\psi_i + e\dot{x}_i A_i - \frac{ie}{2}F_{ij}\psi_i\psi_j + \\ & + \frac{i}{2}e\xi\dot{\xi} + \frac{R^2}{2} - eR\Lambda + ie\xi\psi_j\mathcal{B}_j + \\ & + \frac{e}{2}\epsilon_{ijk}\mathcal{B}_i x_j \dot{x}_k - \frac{ie}{2}\epsilon_{ijk}\mathcal{B}_i\psi_j\psi_k - \frac{ie}{2}\epsilon_{ijk}\psi_r\partial_i\mathcal{B}_r x_j \psi_k \end{aligned} \quad (22)$$

Also from L in (22), we can read the Hamiltonian:

$$\begin{aligned}
H = & \frac{1}{2}(p^i - eA^i)^2 + \frac{ie}{2}\epsilon_{ijk}B_k\psi_i\psi_j + \\
& -\frac{ie}{2}\epsilon_{ijk}\mathcal{B}_i\psi_j\psi_k - \frac{ie}{2}\epsilon_{rji}(\partial_r\mathcal{B}_k)x_j\psi_k\psi_i + ie\mathcal{B}_i\xi\psi_i + \\
& -\frac{e}{2}\epsilon_{jki}\mathcal{B}_jx_k(p_i - eA^i) + \frac{e^2}{8}\epsilon_{jki}\epsilon_{rni}\mathcal{B}_j\mathcal{B}_rx_kx_n + \frac{e^2\Lambda^2}{2};
\end{aligned} \tag{23}$$

in this equation, we have eliminated the component-field without dynamical character.

4.1 The supersymmetric charge

Acting with the supercharge operator (18) on the superfields (17), we can obtain the supersymmetry transformations of the components of the fields; they are:

$$\delta\psi^i = \epsilon\dot{x}^i, \quad \delta\xi = -i\epsilon R, \quad \delta R = -\epsilon\dot{\xi}, \quad \delta x^i = -i\epsilon\psi^i. \tag{24}$$

From the supersymmetric transformations and the Lagrangian (22), we can analytically calculate the supercharge, through the Noether's theorem. The charge operator comes out to be

$$Q = \psi_i\dot{x}_i + \frac{1}{2}(1-i)\xi R - e\xi\Lambda. \tag{25}$$

The supercharge algebra reads

$$[Q, Q]_+ = 2H$$

where H is the Hamiltonian of eq. (23). The quantization was already performed, [20].

This supersymmetry includes both approaches of this model, G-A and L-A, the terms $Psch$ and Mtz that make the correction to the Hamiltonian in L-A, was described in the supersymmetric model. It is possible to give a interpretation about the physical mechanism by which the correction terms $Psch$ and Mtz of the Hamiltonian (9), allow the energy flow from the fifth dimension (extra dimension) to the usual space. And from this new interpretation, we generate this two correction terms $Psch$ and Mtz in the G-A.

5 Extra Electrical Field from the Extra Dimension

The Hamiltonian eq (7) has two correction factors from the extra dimension, fifth dimension, which appear also in the Hamiltonian from the supersymmetry

eq (23), *Psch* term with order of $\frac{A_4}{m}$; and the *Mtz* term with order $\frac{A_4^2}{m}$. Both terms also appear in the Hamiltonian in lower dimensions, which have an extra dimension, as in 1+2 or 1+3, eq (2), (1). The Energy from the fifth dimension only flows to the usual three-dimensional space by the mechanism *Psch*, if the symmetry of the usual space is broken into three simultaneous aspects. It is due to the external electrical vector component (A) on the extra dimension is not homogeneous on the usual space (not constant), generating a preferred direction on the usual space. And the other two factors are due the presence of electrically charged particle with spin $\frac{1}{2}$. By the mechanism *Mtz*, the symmetry of the usual space is broken allowing the energy flow from the extra dimension to the usual space, since exist an electrically charged particle, in presence of a extra electrical field \vec{E} in 3 dimension, or in presence of a pseudo-electrical field \vec{E} in 2 or 1 dimension. Both the extra electrical field and the pseudo-electrical field \vec{E} are generated from the extra-dimension by the existence of a nonzero component of the electrical vector A in the extra dimension, being A breaking the symmetry of space or not. Based on the corrections on the Hamiltonian in the G-A eq (15), in terms of a usual electrical field E , eq (16), we use exactly the same term, which will differ only by a multiplicative constant related to the degree of freedom of the particle, ie the dimension of usual space. In this sense, at the case of 1 +2 dimension, the correction terms of the Hamiltonian in terms of the electrical field is the same given by, MR and OR to the usual electrical fields,

$$MR = \frac{e^2}{32m^3} \mathbf{E}^2 \quad \text{and} \quad OR = \frac{e}{8m^2} \nabla \cdot \mathbf{E} \quad (26)$$

and the definition of the extra electrical field, which is actually a pseudo vector, it is invariant by parity, is

$$E^1 = i4m\Omega\sigma^3\sigma^1 \quad (27)$$

that creates the terms $Psch = \frac{ie}{2m}\sigma^3\sigma^1\vec{\nabla}^1(\Omega)$ and $Mtz = \frac{e^2}{2m}|\Omega|^2$, the proper correction of the Hamiltonian eq (15) in G-A, due to the influence of the energy of the extra dimension. Although the usual electrical field is zero, the extra pseudo electrical field generated by the energy from the extra dimension could rise.

In a general way in 1+2 and 1+3 dimension we can define the pseudo electrical field by,

$$E^i = \frac{i4m}{\sqrt{d}}\Omega\sigma^3\sigma^i, \quad i = \{1, 2\}, d = 2 \text{ to } 1+3 \text{ or } i = \{1\}, d = 1 \text{ to } 1+2 \quad (28)$$

The correction terms of the Hamiltonian on the dimension 1+2, 1+3 and 1+4 in terms of the extra electrical field is similar to MR and OR

$$MR = \frac{e^2}{32m^3} \mathbf{E}^2 \quad \text{and} \quad OR = \sqrt{d} \frac{e}{8m^2} \nabla \cdot \mathbf{E} \quad (29)$$

on 1+3 dimension we have the correction terms $Psch = \frac{ie}{2m}\sigma^3\sigma^i\vec{\nabla}^i(\Omega)$ and $Mtz = \frac{e^2}{2m}|\Omega|^2$. where d is the dimensional degree of freedom of the particle, the

usual space dimension were the particles actually move. In the 1+4 dimention the extra electrical field is really a vector defined by,

$$E^i = -\frac{4m}{\sqrt{3}}\Omega\sigma^i, \quad i = \{1, 2, 3\} \quad (30)$$

that generates the terms $Psch = -\frac{e}{2m}\sigma^i\vec{\nabla}^i(\Omega)$ $i = \{1, 2, 3\}$ and $Mtz = \frac{e^2}{2m}|\Omega|^2$ the proper correction of the Hamiltonian eq (15) in G-A, due to the influence of the energy from the extra fifth dimension. With this correction and conditions the Pauli-Dirac Hamiltonian in G-A is the same one in L-A eq (7), that is a supersymmetric sector of eq (23). So even though the usual electrical field E is zero in the usual three-dimensional space, the existence of a nonzero extra electrical field will allow energy to flow from the fifth dimension to the usual three-dimensional space, by the mechanism $Psch$ and Mtz .

5.1 Conclusion

We showed in two approaches, L-A and G-A the Pauli-Dirac Hamiltonian in 4+1 dimensions, and we performed the supersymmetry of the model inside some conditions. also we could show that the term $Psch$ and the term Mtz arises in a similar way in 1 +2, 1 +3 and 1 +4 dimension, but the extra electrical field associated with these terms in 1 +2 dimensions and 1 +3 are pseudo vectors, only in 1+3 dimension, it will be a vector. The reason for this, is the same by which it was not possible to describe in a 1+2 the supersymmetry here, using the positions of the bidimensional space as bosonic variables and angular momentum, spin as fermionic variables. The angular momentum in 1+2 dimension is not a vector it is a pseudovector, it is invariant by parity, or equivalently one can say that a supersymmetry in this way is not possible here, as was discussed previously by treating the 2 +1 dimension. The extra electrical field E produces the correct terms in eq (8) from the terms MR and OR . The extra electrical field E from the extra dimension was defined in a natural way, leading to a new result, which was not established for the Hamiltonian in G-A. The factor d that appears in the definition of the electrical vector and in the pseudoelectrical vector reveals the dimension where the particle is trapped, it is the usual dimension of space. In graphene the electron for example is apparently confined to a space of dimension two, in this way the third spatial coordinate would be extra. An experiment may show whether this parameter d has in fact physical means, in the case of graphene if actually the electron is confined in a surface, $d = 2$. The existence of an extra vector or extra pseudo-vector type electric is a simple way to see that the space could contain more energy than usually we realize, it would be hidden in other dimensions, and arise when the symmetry of space is broken by some mechanism like $Psch$ or Mtz .

Appendix 1

We quote below the Γ -matrices representing the Clifford algebra of (1+4)D:

$$\Gamma^4 = i\Gamma^5 = \begin{pmatrix} 0 & i_2 \\ i_2 & 0 \end{pmatrix}; \quad \Gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}; \quad \Gamma^0 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}.$$

Appendix

Its adjoint is given by $\bar{\Psi} = \Psi^\dagger \gamma^{tr} = \Psi^\dagger \eta_{(+)}$, where ϑ_+ and ϑ_- are the large and small components respectively, $\eta_{(+)} = \gamma^{tr} = \frac{-i}{\sqrt{2}}(\gamma^s + \gamma^t)$, $\eta_{(-)} = \gamma^{x^4} = \frac{-i}{\sqrt{2}}(\gamma^s - \gamma^t)$ and γ^μ , $\mu = 1, 2, 3, t, s$, are 4×4 matrices that satisfy the Clifford algebra $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$. The representation here for these matrices, is given by

$$\gamma^i = \begin{pmatrix} \sigma^i & 0 \\ 0 & -\sigma^i \end{pmatrix}, \quad \gamma^t = \sqrt{2} \begin{pmatrix} 0 & 0 \\ -\mathbf{1} & 0 \end{pmatrix}, \quad \gamma^s = \sqrt{2} \begin{pmatrix} 0 & \mathbf{1} \\ 0 & 0 \end{pmatrix},$$

where σ^i , $i = 1, 2, 3$, are the Pauli matrices.

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